Design and Analysis of Stochastic Traffic Flow Models for Vehicular Clouds

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Abstract
Intelligent transportation systems (ITS) have attracted an increasing amount of attention within both public and private sectors due to the unprecedented number of vehicles all over the world. ITS aim to provide innovative applications and services relating to traffic management, and enable ease of access to information for various system users. The intent to utilize the excessive on-board resources in the transportation system, along with the latest computing resource management technology in conventional clouds, has cultivated the concept of the Vehicular Cloud. Evolved from Vehicular Networks, the vehicular cloud can be formed by vehicles autonomously, and provides a large number of applications and services that can benefit the entire transportation system, as well as drivers, passengers, and pedestrians. However, due to high traffic mobility, the vehicular cloud is built on dynamic physical resources; as a result, it experiences several inherent challenges, which increase the complexity of its implementations. Having a detailed picture of the number of vehicles, as well as their time of availability in a given region through a model, works as a critical stepping stone for enabling vehicular clouds, as well as any other system involving vehicles moving over the traffic network. Therefore, in this paper we present a comprehensive stochastic analysis of several traffic characteristics related to the implementation of vehicular cloud inside a road segment by adopting proper traffic models. According to the analytical results, we demonstrate the feasibility of running a certain class of applications or services on the vehicular cloud, even for highly dynamic scenarios.

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1. Introduction

Due to the advent of cloud computing and the maturity of its technology, various users are able to access services at any point in time and space. The emergence of a new type of cloud—the vehicular cloud—has been a prominent step forward for intelligent transportation systems. The vehicular cloud is formed autonomously by the traffic on the road, and each vehicle serves as a compute node for the cloud, which offers a great number of benefits not only to drivers, passengers, and pedestrians, but also to the municipal traffic manager and city planners. Those self-organized clouds can work independently, as well as complementing to the conventional cloud structure and providing processing power to deal with the traffic-related issues. This trend is quite promising, due to the sky-high increase in the rate of the number of vehicles on the road. According to US National Transportation Statistics [1], the number of vehicles in American roadways, has reached 253.6 million. Harnessing the excessive computing power of those vehicles has a profound significance for information technology (IT), the economy and society.

At the same time, rapid technological advances in cloud computing [2] provide sufficient technical support for building a dynamic cloud in mobile environments. Cloud can provision resources and services on-demand over the Internet, just like a public utility [3]. Amazon Elastic Compute Cloud (Amazon EC2) is now the largest online retailer to provide dynamic compute capacity in the cloud. On the other hand, an increasing number of businesses prefer to rent their servers, platforms or software in an elastic and scalable manner, instead of purchasing and maintaining them by themselves [4]. The reciprocal benefit accelerates the prosperity of cloud computing, whose main features include pay-as-you-go, virtualization technology, resources on demand, scalability and Quality of Service (QoS). As a result, cloud computing is becoming the main technological trend in IT, and enterprises nowadays invest massive effort in migrating their services to the cloud.

Motivated by the above-mentioned facts and opportunities, the vehicular cloud framework was initially proposed in [5]. In general, various underutilized vehicular resources such as computing power, network connectivity, sensing capability, and storage can be shared with car owners. Moreover,
the aggregated resources can be rented to potential consumers following a specific business model, similar to the traditional cloud infrastructure. However, the most challenging aspect of building such a cloud is dealing with the highly dynamic availability of the vehicles, which distinguishes the vehicular cloud architecture from traditional cloud models.

The autonomous cooperation among vehicles and their “decentralized” management contribute to the complexity and uniqueness of the vehicular cloud. Accurately determining the amount of vehicles and their available time frame is critical for constructing a vehicular cloud. The analysis of the availability of resources in this particular scenario is totally and directly involved with the perception of the traffic flow for a given area, such as a road segment, during a time span. Nevertheless, the number of vehicles varies continually, since the same vehicles move dynamically, with diverse traveling times, behaviors, and speeds.

The main contribution of this work is a detailed stochastic analysis about the distribution of the vehicles and their available time range within a road segment in an urban area. This is a step towards utilizing excessive vehicular resources and constructing the vehicular cloud in a highly dynamic environment. Two separate traffic scenarios, namely free-flow traffic and queueing-up traffic, are modeled and analyzed in this work:

- The macroscopic model described in [6] is drastically extended to represent the free-flow traffic in a roadway segment by obtaining the average number of vehicles and the distribution of the number of vehicles inside the road segment;

- The queueing theory was employed to model the queueing-up traffic. The analytical form of the length of the traffic queue and the waiting time of vehicles are presented. These metrics are essential for properly representing a number of available computation resources and their valid time;

- Based on the models, we carry out a comprehensive analysis of several traffic characteristics related to the implementation of vehicular clouds for both traffic scenarios.

Following this introduction, Section 2 summarizes related work of vehicular clouds. Section 3 provides the mathematical modeling of the study. This section is composed of two subsections; the first focuses on the free-flow traffic scenario, and the second works on the queueing-up traffic scenario. Section 4 demonstrates the analytical results, and presents the detailed anal-
ysis of each scenario. Finally, Section 5 concludes the results and describes our future directions of research.

2. Related Works

In order to enable access to rich services and applications, modern vehicles are equipped with reasonably powerful computational capabilities. Cloud computing extends these capabilities by featuring limitless resources and enhancing provision of services. Also, the vehicles’ built-in resources are likely to be underutilized most of the time, such as when they are in a parking lot or traffic jam. There is a great potential in employing each vehicle’s unused computing resources to build clouds autonomously. Enabling vehicles to connect and participate in a vehicular cloud is an extremely valuable strategy for solving complex issues in real-time and in loco, as it is considered a paradigm shift in transportation systems [7].

The issues faced in the dynamic interconnection of vehicles to assemble a vehicular cloud is closely related to the challenges faced in vehicular networks [8]. Both the vehicular cloud and vehicular networks provide a means for the design of solutions in Intelligent Transportation Systems. Vehicular networks can be seen as an expansion of Mobile Ad-hoc Networks, which have shown a steady increase in popularity with advancements on solutions, technologies, and an extensive range of applications it can support. Vehicular networks basically present two major architectures: Vehicle to Infrastructure communication (V2I) and Vehicle to Vehicle communication (V2V) [9]. Initially, considerable work has been applied to vehicular networks, focusing on avoiding traffic hazards or unpleasant driving experiences [10]; these types of issues in transportation require overcoming challenges in efficiently gathering and disseminating real-time data [11]. Vehicular networks have now expanded this scope to include concerns regarding safety, entertainment, privacy, and security.

Vehicular cloud is a novel field that was initiated with the advent of Cloud computing, more specifically mobile Cloud [12, 5, 7]. As a result, the works in this field present taxonomies, discuss challenges, define a class of applications, describe possible architectures, and determine limitations on implementation [13]. There is significant consideration on applications and services enabled by vehicular clouds [7]. Some works reason on primary services that are empowered by vehicular cloud, and classify them as Computation as a Service [14, 15, 16], Network as a Service (NaaS) [17, 18], Storage as a Service (STaaS) [14, 19, 20], and Cooperation as a Service (CaaS) [21, 22].
Focusing on a class of application with resource provisioning for these primary services in mind, this class can be classified in two major categories: static vehicular cloudlets and highly-dynamic vehicular clouds [23]. Work has been conducted to build a data center with resources available from vehicles in a parking lot [14]. This work assumes a static environment, since its context consists of estimating the amount of time vehicles rest in the long-term parking lot of an airport. This work extends the traditional cloud by estimating the amount of resources available through a stochastic process with time-varying rates for vehicles’ departure and arrival. Another work has been proposed in [22] to facilitate the development of an intelligent parking system. The defined services for parking lot management employ approaches on the conventional cloud and the vehicular cloud.

The category of applications described above aims to build a cloud in highly-mobile environments. A work on a dynamic traffic management system has been conceptually performed in [5]. This work suggests using the computational power of vehicles at rest in the road segment where there is a traffic jam to identify solutions that can mitigate congestion. Therefore, to the best of our knowledge, there is no work that provides a concrete design or implementation of a dynamic vehicular cloud; we therefore define a flow model in this work that facilitates an estimation of the number of vehicles in a road segment. This estimation provides tools to determine the feasibility of designs in real transportation environments, enabling us to assess the amount of resources and their availability.

The challenges and motivation for this work are summarized as follows:

- Most of the works in vehicular cloud restrict themselves on the taxonomy definition level, currently dealing with a rather stable environment, such as parking lots, for the implementation of a VC;
- There is no implementation of VC on the road, but the majority of vehicles spends a substantial amount of time on the road;
- It is imperative to develop a model that can be used to evaluate a number of computing resources in a road segment;
- The model must essentially provide a proper range of parameters for a VC simulation and must envision a VC involving the dynamics of roads, accurately supporting related applications.
Figure 1: Vehicular Cloud Resource Management Layers

3. Stochastic Modeling

Figure 1 shows the brief architecture of the vehicular cloud in terms of the utilization of resources. Unlike the conventional cloud infrastructure, the physical resource layer of the vehicular cloud is dynamic. Therefore it is vital for the resource manager to predict the amount of available computational resources in order to provide services to the upper layers. Two common traffic scenarios are modeled in this work: free-flow traffic and queueing-up traffic.

3.1. Free-flow Model

In the free-flow traffic model, the traffic is not blocked by any obstacles such as traffic light, stop sign, bifurcations or traffic congestion, and a driver can drive at any speed, provided they remain within road constraints. Therefore, the traffic density in this case is more likely to be low or medium, as high density slows down the traffic due to the road capacity. In other words, vehicles are vastly isolated on the road and the arrivals at the entry point of the road segment are independently and identically distributed. The Poisson process is a common random process for depicting vehicles’ arrival. We use the free-flow model [6] as a base to carry out our analysis.

Figure 2 shows a typical roadway segment [SD]. The traffic theory in [24] represents the average speed observed over a road segment, and is given by:

\[ \bar{V} = V_{\text{max}}(1 - \frac{\rho_v}{\rho_{\text{max}}}) \]  

where \( \bar{V} \) is the average speed, \( V_{\text{max}} \) is the upper speed limit, \( \rho_v \) is traffic density (Unit: \text{veh/meter}) and \( \rho_{\text{max}} \) is the maximum traffic density. In general, each road segment has a speed range \([V_{\text{min}}, V_{\text{max}}]\) and the traffic model in [6] assumes that vehicles’ speeds are a Gaussian distribution between \( V_{\text{min}} \) and \( V_{\text{max}} \) with mean \( \bar{V} \) and standard deviation \( \sigma_V \). Another assumption is
that vehicles maintain the same speeds over the length of the road segment $L_{SD}$. Therefore, the navigation time from arrival reference point $S$ to departure reference point $D$ for an arbitrary car $i$ with speed $v_i$ is $T_i = \frac{L_{SD}}{v_i}$, which is commonly called residence time for a vehicle. It is also a random variable due to the arbitrary value of $v_i$. Let $F_T(\tau)$ denote the cumulative distribution function (CDF) of the residence time.

\[
F_T(\tau) = P[t \leq \tau] = P[\frac{L_{SD}}{\nu} \leq \tau] = P[\nu \geq \frac{L_{SD}}{\tau}] = 1 - F_V(\frac{L_{SD}}{\tau}) \tag{2}
\]

where $F_V(\nu)$ is the CDF of vehicular speed. Accordingly, the probability density function (PDF) of the residence time is shown as

\[
f_T(t) = \frac{dF_T(\tau)}{d\tau} = \frac{M \cdot L_{SD}}{\tau^2 \nu \sqrt{2\pi}} e^{-\left(\frac{L_{SD} - \bar{\nu}\tau}{\nu \sqrt{2}\tau}\right)^2} \tag{3}
\]

where $M$ is a normalization factor, as defined in [6].

The real free-flow vehicular traffic was investigated in [25], where the real-time traffic measurements collected from several highways in the City of Madrid are analyzed. A Gaussian-exponential mixture model was proposed in order to characterize the time intervals between vehicles on the highway. The results also reveal that when the traffic density is a small or medium number, the vehicles are somehow isolated, and the time intervals between two consecutive vehicle arrivals, i.e. inter-arrival times, feature the exponential distribution. As a result, the PDF for the inter-arrival time is expressed as

\[
f_A(t) = \mu e^{-\mu t} \tag{4}
\]

in which the mean value of inter-arrival time $1/\mu$ is inversely proportional to arrival rate $\mu$ (Unit: veh/s). Consequently, the CDF of the inter-arrival time is

\[
F_A(\tau) = P[t \leq \tau] = \int_{-\infty}^{\tau} f_A(t) \, dt = 1 - e^{-\mu \tau} \tag{5}
\]
Figure 3a and 3b demonstrate the stochastic features of a Poisson arrival process. Figure 3a presents the probability of vehicular occurrences in a 15-minute interval. Specifically, if the traffic arrival rate $\mu = 0.2$ vehicles/minute, 2 or 3 vehicles are expected to show up at the arrival reference point during the period of 15 minutes, with a probability rate of 23%. On the other hand, when $\mu = 0.5$ vehicles/minute, it is expected that more vehicles will enter the road segment. For instance, in contrast to 10% probability for the event that 10 vehicular arrivals within 15 minutes when $\mu = 0.5$, that probability is negligible when $\mu = 0.2$. Figure 3b depicts the exponentially distributed inter-arrival time of a Poisson process. For example, the probability of the inter-arrival time that exceeds 4 minutes is 45% when $\mu = 0.2$ vehicles/minute; this value plummets to 12% as $\mu$ increases to 0.5 vehicles per minute. In addition, the chance of an inter-arrival time that is longer than 12 minutes is zero when $\mu = 0.5$ vehicles/minute, which means the next arrival will occur within 12 minutes, with approximately 100% probability.

The vehicular arrival in free-flow traffic is commonly modeled as a Poisson process; therefore, Little’s Formula [26] can be applied here to estimate the average number of vehicles inside the segment [SD].

$$E(N) = \mu E(T) = \mu \int_0^\infty t f_T(t) \, dt = \mu \int_0^\infty \frac{M \cdot L_{SD}}{L_{SD} - V} \cdot \frac{L_{SD} - V}{\sigma V \sqrt{2\pi}} e^{-\left(\frac{L_{SD} - V}{\sigma V \sqrt{2\pi}}\right)^2} \, dt \quad (6)$$

Normally, the first moment (i.e. mean value) of a random variable offers very limited information about the actual distribution of this variable. Higher moments are required to obtain a full picture. Likewise, to have a detailed and precise description of the available computing capabilities inside the road segment, it is necessary to characterize the dynamics of the
vehicles. A counting process \( \{ N(t) | t \geq 0 \} \) with time parameter \( t \) can be used to model the number of vehicles. The event \( \{ N(t) = i \} \) represents segment \([SD]\) holding \( i \) vehicles at time \( t \). The probability of this event is denoted as \( P_i(t) = P\{N(t) = i | t \geq 0\} \). Assume the total number of vehicular arrivals during time interval \((0, t)\) is \( s \), and \( p(t) \) is the probability that a vehicle remains in \([SD]\) at time \( t \); the distribution of vehicles within segment \([SD]\) can be derived by the theorem of total probability.

\[
P_n(t) = \sum_{s=0}^{\infty} P_{n|s}(t) \cdot M_s(t)
\]

where \( P_{n|s}(t) \) denotes the probability of \( n \) vehicles remaining in \([SD]\) at time \( t \) given \( s \) vehicular arrivals during \((0, t)\), while \( M_s(t) \) represents the probability of an event that \( s \) vehicular arrivals in time interval \((0, t)\).

Additionally, an arbitrary vehicle might arrive at segment \([SD]\) at time \( \tau \) \((0 < \tau \leq t)\). The equivalent event of observing this vehicle in \([SD]\) at time \( t \) is that the residence time of the vehicle is greater than \( t - \tau \). Equation \( 8 \) shows the probability of the equivalent event.

\[
P(u > t - \tau) = 1 - P(u \leq t - \tau) = 1 - F_T(t - \tau)
\]

where \( u \) denotes the residence time of a random vehicle. Therefore, we can obtain the limiting behavior of \( t \cdot p(t) \) by assuming \( t \to \infty \), as described in Equation \( 9 \)

\[
t \cdot p(t) = \int_{0}^{\infty} [1 - F_T(t - \tau)] \cdot \frac{1}{t} \, d\tau = \int_{0}^{\infty} [1 - F_T(\tau)] \, d\tau
\]

Since we assume that vehicular arrival is a Poisson process, given a specific time interval, the arrival times are independently and uniformly distributed during this time period. In our case, the PDF of arrivals during \((0, t)\) is \( 1/t \), which has been employed in Equation \( 9 \). The closed-form expression of \( P_n \), which represents the probability mass function (PMF) of the number of vehicles within \([SD]\), can be obtained by substituting Equation \( 9 \) into Equation \( 7 \); the result is shown in Equation \( 10 \)

\[
P_n = \frac{[\mu E(T)]^n e^{-\mu E(T)}}{n!}
\]
where $E(T)$ is the mean value of residence time within segment $[SD]$.

It is evident that the number of vehicles in roadway segment $[SD]$ over the long term is a Poisson-distributed random variable with mean value $\mu E(T)$ according to Equation 10. Moreover, this distribution is independent of the vehicular velocities; in other words, the number of vehicles remains as a Poisson-distributed random variable, regardless of which velocity distribution is chosen to represent the traffic mobility within segment $[SD]$. The distribution in Equation 10 matches the results in [27], where the expected number of vehicles in segment $[SD]$ is discussed. Moreover, the limiting behavior of the variance of the counting process $\{N(t) | t \geq 0\}$ is expressed in Equation 11.

$$\text{VAR}(N) = \mu E(T)$$

It is noteworthy that variance $\text{VAR}(N)$ has the same form as mean value $E(N)$ in [27], which implies the percent variance is as high as 100%. However, in order to prove the feasibility of the vehicular cloud and utilize vehicles’ excessive computing power, it is critical to demonstrate that a scenario very few vehicles exist in $[SD]$ is unlikely to occur, despite the high percent variance.

### 3.2. Queueing-up Model

Urban roads are more likely to be congested, due to a rapidly increasing number of vehicles and the traffic capacities of city roads. Figure 4 illuminates a normal traffic queue scenario. The bottleneck of the traffic could be an intersection, construction site or location of an accident. In general, the queue is bound to emerge when the departure rate is less than the arrival rate.

A few assumptions are made before we conduct our analysis. The roadway segment has only one lane, and the vehicular arrival follows a Poisson process with parameter $\lambda$, which means the inter-arrival time is an exponential distributed random variable with parameter $1/\lambda$ [26]. In addition, we define the time it takes a vehicle to pass the bottleneck as the leaving time.
The total delay time $D_i$ of an arbitrary vehicle $i$ in a roadway segment is the summation of its waiting time $W_i$ in the queue and the leaving time $\tau_i$, as described in [12]

$$D_i = W_i + \tau_i$$  \hspace{1cm} (12)

We consider the queue with some level of equilibrium, which means the initial conditions of this queue have faded out and the state probabilities are independent of the preconditions. Therefore, the expected length of the queue maintains a constant value. We defined traffic density $\rho$ as the expected number of vehicular arrivals $E(V)$ within the time period of $E(\tau)$, which represents the expected value of the leaving time. Moreover, as the arrival process is Poissonian with rate $\lambda$, the vehicular arrival is a Poisson distributed random variable, with a mean value of $\lambda \tau$ for a given time range $\tau$.

$$\rho = E(V) = \lambda E(\tau)$$  \hspace{1cm} (13)

Since the expected value and the variance have the same form for a Poisson distribution, the second moment of the arrival process $V$ is

$$E(V^2) = \text{VAR}(V) + E^2(V) = \lambda \tau + (\lambda \tau)^2$$  \hspace{1cm} (14)

According to Little’s law, Equation [13] and [14] as well as the mathematical deduction in [28] [29], the expected number of vehicles in the queueing system is

$$E(N) = \lambda E(W + \tau) = \rho + \frac{\lambda^2 \sigma^2 + \rho^2}{2(1 - \rho)}$$  \hspace{1cm} (15)

in which $\sigma^2$ represents the standard deviation of vehicular leaving time. We can easily derive the mean value of leaving time $E(\tau) = \rho/\lambda$ by utilizing Equation [13], then substitute $E(\tau)$ in [15] with $\rho/\lambda$. Eventually, we have the expected waiting time for a vehicle

$$E(W) = \frac{1}{\lambda}(E(N) - \lambda E(\tau)) = \frac{\lambda^2 \sigma^2 + \rho^2}{2\lambda(1 - \rho)}$$  \hspace{1cm} (16)

Consequently, the expected traffic queue length is

$$E(Q) = \lambda E(W) = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1 - \rho)}$$  \hspace{1cm} (17)

Two specific scenarios are considered here.

- The leaving time is fixed. The standard variance for the leaving time is $\sigma^2_{\tau} = 0$ in this case, so the Equation [17] becomes

$$E(Q) = \frac{\rho^2}{2(1 - \rho)}$$  \hspace{1cm} (18)
The leaving time is an exponentially distributed random variable. As a result, we have $\sigma_\tau^2 = E^2(\tau) = \rho^2/\lambda^2$, substituting in Equation 17 thus gives

$$E(Q) = \frac{\rho^2}{1 - \rho}$$

(19)

Utilizing the computing resources of vehicles in the queue requires more detailed information about the distribution of the queue length and waiting time, which represents the amount of resources and their time availability, respectively. The Pollackzek-Khinchin transform Equation is used to derive the closed-form of the distribution; however, the numerical methods are employed when the closed-form is not available.

$$G_N(z) = \frac{(1 - \rho)(z - 1)b^*(\lambda(1 - z))}{z - b^*(\lambda(1 - z))}$$

(20)

where $G_N(z) = \sum_{k=0}^{\infty} Pr[N = k]z^k$ represents the generating function for $N$ and $b^*(s)$ denotes Laplace transform of the PDF of leaving time. When the vehicular leaving time is an exponentially distributed random variable with mean value $1/\mu$, then the PDF of leaving time is $f_\tau(t) = \mu e^{-\mu t}$. Consequently, the Laplace transform of $f_\tau(t)$ is

$$b^*(s) = \int_0^{\infty} e^{-st}f_\tau(t) \, dt = \frac{\mu}{s + \mu}$$

(21)

Substituting $b^*(s)$ in Equation 20 with Equation 21, we can get the expression of $G_N(z)$ when the leaving time is exponentially distributed.

$$G_N(z) = \frac{(1 - \rho)(z - 1)\mu}{z - \mu(\lambda + 1)} = \frac{1 - \rho}{1 - \rho z} = \sum_{n=0}^{\infty} (1 - \rho)\rho^n \cdot z^n$$

(22)

where we used the steady state condition ($\rho < 1$ and $|z| < 1$). Equation 22 implies the PMF of the number of vehicles $N(t)$ in the queue is

$$p_n = (1 - \rho)\rho^n \quad (n \geq 0)$$

(23)

Similarly, the PDF of the waiting time can be derived by using the Pollackzek-Khinchin transform Equation 24

$$c^*(s) = \frac{(1 - \rho)s}{s - \lambda + \lambda b^*(s)}$$

(24)

where $c^*(s)$ represents the Laplace transform of the waiting time PDF $f_W(w)$. Replacing $b^*(s)$ with Equation 21 in Equation 24 we have

$$c^*(s) = (1 - \rho)\frac{s + \mu}{s + \mu - \lambda} = (1 - \rho)\left(1 + \frac{\lambda}{s + \mu - \lambda}\right)$$

(25)
Then we obtain the PDF of the waiting time by implementing the reverse Laplace transform of Equation 25

\[ f_W(w) = (1 - \rho)\delta(w) + \rho(\mu - \lambda)e^{-\lambda w} w > 0 \quad (26) \]

It is noteworthy that the delta function at \( w = 0 \) corresponds to the fact that an arbitrary vehicle has zero waiting time in the queue with probability \( 1 - \rho \).

In addition, we focus on the second scenario, in which a vehicle's leaving time is deterministic, and assume the leaving rate has a fixed value of 1 vehicle/second. Therefore, the traffic intensity \( \rho = \frac{\lambda}{\mu} = \lambda \). By Equation 20, we have

\[ G_N(z) = \frac{(1 - \lambda)(z - 1)\exp(\lambda(z - 1))}{z - \exp(\lambda(z - 1))} \quad (27) \]

in which we used the mathematical fact that the PDF of a constant random variable (i.e. degenerate distribution) is a delta function at \( t = 1 \) s (first vehicle always leaves at 1s).

The explicit expression of PMF of \( N(t) \) is obtained by the Taylor expansion of Equation 27 according to the deduction process in [30].

\[
\begin{align*}
Pr[N = 0] &= 1 - \lambda \\
Pr[N = 1] &= (1 - \lambda)(e^\lambda - 1) \\
Pr[N = 2] &= (1 - \lambda) \left( e^{n\lambda} + \sum_{j=1}^{n-1} e^{j\lambda}(-1)^{n-j} \cdot \left[ \frac{(j\lambda)^{n-j}}{(n-j)!} \cdot \frac{(j\lambda)^{n-j-1}}{(n-j-1)!} \right] \right), n \geq 2
\end{align*}
\]

Similarly, the waiting time distribution can also be determined by using Equation 24 when the leaving time is a constant value.

\[ c^*(s) = \frac{(1 - \lambda)s}{s - \lambda + \lambda e^{-s}} \quad (29) \]

where we assume the leaving rate \( \mu = 1 \) vehicle/second and consequently, the Laplace transform of leaving time becomes \( b^*(s) = e^{-s} \) for the reason that the PDF of the leaving time is a delta function \( \delta(t-1) \) in this scenario. It might be hard to obtain the explicit form of the PDF of the waiting time from Equation 29. However, the numerical analysis can be achieved by implementing the fast Fourier transform methods on Equation 29.

### 4. Analytical Results

From the models described in Section 3, we present a comprehensive discussion of the results obtained from our analysis on the two traffic scenarios: free-flow and queueing-up traffic. The previous works are not analytically
evaluated in this work due to the fact that all available VC resource prediction implementations focus on relatively stable environments, such as a parking lot, which is not similar to the scenarios we discuss here in this work, where the highly mobile environments are analyzed. The analysis of the models considers different metrics and factors that might condition or restrict vehicle traffic flows. In order to observe the models conditioned under different situations, the models’ parameters have been configured according to the value ranges described in Table 1.

In this study, traffic load and vehicles’ residence times are the major factors we observe in our scenarios since the scope of this work concentrates on the utilization of excessive, under-utilized resources of vehicles in a road segment. Traffic load consists of the quantity of resources that can be harvested from vehicles, while residence time determines the length of time resources are available.

4.1. Free-flow Model

By essence, the free-flow traffic model deals with low traffic density, with relatively high average speeds and low residence time. Even though this model represents a scenario in which it is impractical to employ computational resources because of the rapidly changing topology and sparcity of the vehicular network, it can provide beneficial solutions for intermittently-

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol &amp; Units</th>
<th>Default Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic Density</td>
<td>( \rho ) (veh/meter)</td>
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</tr>
<tr>
<td>Max Traffic Density</td>
<td>( \rho_{\text{max}} ) (veh/meter)</td>
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</tr>
<tr>
<td>Road Segment Length</td>
<td>( L_{SD} ) (meter)</td>
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<tr>
<td>Maximum Speed</td>
<td>( V_{\text{max}} ) (m/s)</td>
<td>50</td>
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</table>
connected vehicular networks. An integration of this model with delay-tolerant mechanisms might serve and fit requirements to increase the packet delivery ratio.

4.1.1. Traffic Load

The relation between the analysis parameters and average number of vehicles within a road segment [SD] has been investigated as described in Equation 6 to provide a study on traffic load.

As depicted in Figure 5, the average number of vehicles within a road segment [SD] increases with the vehicular density growth. This behavior is conditioned through the average vehicle speed ($\bar{V}$), which decreases as $\rho_v$ increases; consequently, the flow of vehicles also increases proportionally. Therefore, the arrival of vehicles in a road segment develops with growing frequency under such conditions.

Figure 6 provides analytical values that compare the average number of vehicles in a given road segment with its length. The chances of finding more vehicles in a road segment are directly proportional to the dimension of the segment. The figure shows that there is a linear association in the model that connects the quantity of vehicles and the segment length. From the analysis, the average vehicle residence time is longer in a longer road, when the speed distribution is the same. With the longer time required to traverse the road segment, the average number of vehicles within the segment increases proportionally, according to Little’s law.

Figure 7 presents an analysis of the free-flow model on vehicle speed. Results show that the average number of vehicles in a road segment grows as the maximum vehicle speed in the segment increases from 10 meters per second (36 km/h) to 60 meters per second (216 km/h); however, this behavior oc-
curs at initial maximum speeds, and the number of vehicles asymptotically converges as the maximum surpasses a threshold. As described in Equation 1, the maximum speed directly influences the average vehicle speed, and a higher maximum speed consequently allows for an increase in the vehicle arrival rate $\mu$. The average number of vehicles $E(N)$ also increases, as defined in Equation 6. Nevertheless, high vehicle speeds cause the average resident time $E(T)$ to decrease due to the limited, fixed road length. Finally, the decrease of average resident time neutralizes the increase of the vehicle arrival rate.

When the vehicle density in the road segment increases, the probability $Pr[N(t) < E(N)]$ converges to 0.5, based on Equation 10. This behavior is shown in Figure 8, which depicts that this probability tendency is expected, since both $VAR(N)$ and $E(N)$ equal $\mu E(T)$ for the defined Poisson process. Let vehicle density in a road segment of $\rho = 0.02$ (vehicles/meter); the probability of containing a number of vehicles within $[SD]$ less than half of the expected value is 0.08. Assuming an increase in vehicle density ($\rho$), the expected value quickly tends to 0. For example, the probability $Pr[N(t) < 6]$ is equal to 0.02 if the expected value of $N(t)$ is defined as 12 with a vehicle density of $\rho = 0.04$. These conditions ensure that there are more than 5 vehicles in the road segment $[SD]$ with a probability of 0.98. The value of probability $Pr[N(t) < E(N)/3]$ stands near zero for any vehicle density in the road segment, indicating that the probability of occurring at least $E(N)/3$ vehicles in the segment is 100%. This assures us that there are at least $E(N)/3$ vehicles within $[SD]$ at any time for utilization of any application.

The analysis of the number of vehicles in a road segment $[SD]$ is depicted in Figure 9. For instance, there is a probability of $Pr[N(t) > 7] > 0.99$ in
encountering at least 7 vehicles in the segment, as the vehicle density equals $\rho = 0.06$. Furthermore, increasing the probability to 0.80, it is possible to find at least 12 vehicles in the segment. With a probability of 0.5 on the events, more than 15 vehicles are in road segment $[SD]$. 

4.1.2. Residence Time

The residence time contains valuable information for vehicular clouds, since it estimates the time availability of a vehicle, as well as its computational resources, in a road segment. Estimating the availability of resources is essential for properly scheduling cloud tasks. In the context of free-flow traffic, the fixed road length and an increasing maximum speed causes the average residence time $E(T)$ to decrease, as described in Figure 10. As the average residence time decreases, the average number of vehicles in the road segment decreases; this behavior is depicted in Figure 7.
4.2. Queuing-up Model

Vehicles queueing-up in a road segment facilitates many promising applications, due to longer residence times and higher vehicle density in a road segment. For instance, vehicles under these circumstances can autonomously self-organize and dynamically build a vehicular cloud, which can help to solve traffic-related problems in real-time. Such a cloud can execute traffic management applications and dynamically and accordingly extend green lights in a given direction, for the sake of alleviating traffic congestion.

4.2.1. Traffic Load

The average number of vehicles in a road segment increases in accordance with the intensity of the traffic. This behavior follows the queueing-up model described in Equation 18 and 19 and is shown in Figure 11. The average number of vehicles increases to a large value in the event that the traffic intensity is close to 1, representing an intense traffic jam in the segment. For instance, the average number of vehicles in the queue is $E(Q) = 0.8^2/(1 - 0.8) = 3.2$ when the leaving time is exponentially distributed and with $\rho = 0.8$. Following this relation, $\rho = 0.88$ leads to an average number of vehicles of $E(Q) = 6.4$. Consequently, an increase of 0.08/0.8 = 10% in the arrival rate leads to a 100% increase in the mean number of queueing vehicles.

Equation 23 provides PMF of the number of vehicles in the queue, considering that the leaving time is exponentially distributed, and including the departing time vehicle in the segment. Figure 12a depicts the behavior of the PMF in the queueing-up model, in which we focus on achieving an equilibrium scenario that satisfies the condition $\rho = \frac{\lambda}{\mu} < 1$. As described in the figure, the traffic queue shows a lower chance to occur, close to 0, as the
departure rate of vehicles is twice the size of the arrival rate, represented by \( \rho = 0.5 \). The PMF also shows a uniform distribution, as the arrival rate is close to the departure rate \( (\rho = 0.99) \). This demonstrates that the number of vehicles can become very large in the presence of probability such as a situation with a low number of queued vehicles. Figure 12b indicates that the probability of the number of queueing vehicles is conditioned by a given number. This demonstrates that it is very unlikely that more than 5 vehicles will be queued with \( \rho = 0.5 \). The same close-to-zero chance is observed with 13 vehicles, \( P[N(t) > 13] \), with \( \rho = 0.75 \). However, encountering more than 20 vehicles in the queue occurs with a probability of 0.82 with \( \rho = 0.99 \). This shows a high chance of forming a queue in the road segment, increasing the chances that resources might be utilized.

In the scenario in which the leaving time of vehicles is constant, different distributions are obtained, as depicted in Figures 13a and 13b. Compared with a scenario that shows the exponentially distributed leaving time of
vehicles, the fixed leaving time matches some aspects; for instance, with low \( \rho \), the PMF and probability curves follow the same pattern, showing similar values. The difference resides on \( \rho = 0.99 \); in this case, the probability of encountering more than 20 vehicles in the road segment is 68% for a fixed leaving time. This presents a lower value when compared with the exponentially distributed leaving time scenario, which shows a chance of 82% under the same conditions. This difference is justified in Figure 11, which shows that an exponentially distributed leaving time scenario presents a larger number of vehicles in the segment than a fixed leaving time scenario does.

### 4.2.2. Residence Time

The residence time is a significant metric that provides a means of determining the amount of time computing resources are available in the road segment, in order to schedule and migrate applications and tasks. Figure 14 presents the average waiting times of vehicles in a traffic queue of our exponentially distributed and fixed leaving time scenarios with \( \rho = 0.8, 0.9 \).

Figure 15a depicts the PDF of the waiting times of vehicles in a queue with an arrival rate of \( \mu = 1 \), and the leaving time of vehicles follows a random variable with exponential distribution. The analytical results totally match the behavior of the PMF shown in Figure 12a with \( \rho = 0.5 \), which means that 50% of the vehicles traverse the road segment without waiting in the queue. With \( \rho = 0.99 \), it is observed that the waiting time of vehicles is uniformly distributed over a long time frame. Larger wait times occur due to arrival rates close proximity to departure rates. The times are better described in Figure 15b which contains probabilities of waiting times according to different arrival rates. For a \( \rho = 0.5 \), it is very unlikely that
any vehicle will wait in the queue for longer than 8 seconds. On the other hand, with $\rho = 0.99$, there is a probability of 0.75 that it will take longer than 30 seconds for vehicles to traverse the road segment.

Finally, waiting times in the queue according to the fixed leaving time scenario are depicted by Figures 16a and 16b. Consistent with the results presented in Figure 14, the fixed leaving time scenario describes shorter leaving times when compared with the exponentially distributed leaving time scenario.

5. Conclusion

In this paper, we have described two traffic flow models to represent the behavior of vehicles in a road segment. The models provide the analytical means for dynamically estimating the availability of computational resources in the traffic network, and can integrate a vehicular cloud. The proposed models reflect two common scenarios: free-flow and queueing-up traffic. Different from the previous works that explore such resources in a
static environment, like the long-term resting of vehicles in an airport parking lot, this work observes a rather dynamic environment, with a limited residence time of vehicles in a road segment, lasting for a matter of seconds or minutes. The estimates allow the design of models to determine the feasibility and possible scenarios for assigning computational tasks to vehicles, as well as migrating tasks from departing vehicles to those arriving in a road segment. Cluster computing with mobile nodes and message passing interface [31] are approaches for resolving assignment issues.

Our future work consists of implementing our free-flow traffic model and evaluating it through simulators, such as NS2 [32] or OMNeT++ [33], to observe the distribution of computational tasks on vehicles in a road segment and the effect of load migrations among vehicles. In a less dynamic scenario, the queueing-up traffic situation, we intend to couple our model with scheduling techniques to determine the best task assignment for benefiting applications. In the context of unexpected critical traffic issues, it is ideal that processing deadlines are matched with the time span of an on-demand assembled vehicular cloud. Thus, critical applications spawned for solving critical issues are able to conclude their execution, resulting in a valuable answer. Moreover, our work explored the traffic flow in single-lane road segments in which an equilibrium is achieved. This scenario is expected to be extended into multi-lane road segments, involving non-equilibrium queues.

References


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