MATH 1P40 Winter 2020 REVISED
Assignment #3

Due: Saturday March 28 by 16:00

(virtual extra help hours TBA)

Your goal is to write a program that allows a user to first input a particular cubic equation and then explore its dynamics on the interval [0,1]. This assignment is to be done in pairs.

Part I: Your interactive dynamical system exploration program

1) The user should be able to enter the parameters a and b for the function $g(x)=-2x^3 + 3(a+b)x^2 - 6abx + 4$

2) Your program should (internally) find the exact maximum M and minimum m of the function g(x) on the interval [0,1]. *(Hint: First by hand or with Maple, use the closed interval method and carefully consider the critical points. You then code your results in vb.net)*

3) Set f(x) = (g(x)-m)/(M-m). Note that the range of f(x) is exactly [0,1]. At the click of a button, we see the graphs of y=f(x) and y=x appear. (These graphs should just touch the bottom and top of your picture box.) **(20 marks)**

4) The user should be able to enter an initial value for the dynamical system determined by f and at the click of another button see a table of values appear and the dynamics (cobweb) drawn in the picture box as in parts 6) to 11) of lab#9. **(25 marks)**

Your program should have an attractive user-friendly interface and good programming style: it should use comments, functions and sub procedures, and should be efficient. **(15 marks)**

Part II: The exploration and hand-written (or typed) report. Your hard-copy report will consist of four parts under the following headings:

1. INTRODUCTION. Write a short paragraph introducing your project. If you use resources (internet, book, article, etc.), give the reference(s) — up to 8 lines **(2 marks)**

2. MATHEMATICS BACKGROUND USING AN EXAMPLE — up to 2 pages **(8 marks)**
   a. Use the two last digits, d_1 and d_2, of your student number and set the values $a=d_1/10$ and $b=d_2/10$; this defines a specific function g. Use it in the following.
   b. Using any technology (e.g. Maple), draw the graph of g with domain [0,1].
   c. Find the maximum and minimum of g, and define f as in step 3 (Part I).

d. Using any technology (e.g. Maple), draw the graph of f, and write a sentence or two to explain its relation to the graph of g.
e. Select an initial values x_0, use your program to compute the first 10 terms of the sequence of the iterative function system based on f, and use the data to explain how the sequence is built. Identify the convergence or divergence of the sequence.
f. Draw manually the corresponding cob-web (in the graph of f), and describe how the convergence or divergence of the sequence is visualized.
g. Show how to find (algebraically) the fixed points of f (you may use Maple for computations), and plot them in the graph of f. Using your program, classify them (attracting, repelling or neither) and describe in your own words what each classification means.

3. DATA COLLECTION ABOUT INTERESTING CASES— up to half a page
   a. Use your program to find values of a and b so that f(x) has three fixed points. Use any method (including Maple) to prove that they are fixed points. Classify each point as attracting or repelling or neither and give written evidence for your claims. **(20 marks)**
   b. Find 3 different pairs of values of a and b and a starting value so that subsequent values oscillate closer and closer to a finite number (between 3 and 100) of values. Describe what happens. **(7 marks)**

4. DISCUSSION/CONCLUSION. Write a short paragraph concluding your exploratory work (e.g., discuss further about 3a) or 3b); about dynamical systems; about the use of cobwebs, etc). If you use resources (internet, book, article, etc.), give the reference(s) — up to half a page **(3 marks)**